PISM and Ice Dynamics

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Abstract

My research project for this summer consisted of testing the new ice model, Parallel Ice Sheet Model (PISM), developed at the University of Alaska by Ed Bueler and Jed Brown. Here I discuss some fundamental concepts of ice dynamics and glacier mechanics. Moreover, I analyze the governing equations that describe glaciers through space and time and introduce the numerical model, PISM. PISM is used to solve these equations and thus model large-scale glacier flow over long periods of time and glacier response to changes in initial and boundary conditions. We run PISM through a series of benchmark simplified geometry experiments as well as some real data and a discussion of the model output follows. These results, as predicted, follow the basic concepts of ice dynamics.
Acknowledgements

I would like to thank Dean Dan Stein and the Courant Institute for giving me with the opportunity and financial support to work on this project. Thank you Professor David Holland and Karen Chang for guiding me through my project and answering my many questions. Also thank you to Chris Grim of ITS for the support he provided me with using Max. I would also like to thank Ed Bueler and Jed Brown at the University of Alaska, the PISM creators, for providing the code and for the very helpful suggestions they gave me.

Introduction to ice dynamics

Glaciers can be classified by shape or by thermal characteristics, and these classification aids in describing them and understanding their properties. By shape glaciers are divided into the following classes: valley glaciers, which are long, narrow and flow in one direction, down a valley. Tidewater glaciers are valley glaciers which reach the coast and interact with the sea. Cirque glaciers are very short valley glaciers, which occupy only a small mountain basin. In contrast to cirque glaciers, ice caps spread out in all directions from a central dome. Ice sheets are large ice caps and lastly outlet glaciers are valley glaciers flowing outward from an ice cap or ice sheet.

In terms of thermal characteristics, glaciers are classified as: polar glaciers, whose temperature is below the melting temperature of ice everywhere, except maybe at the bed. Polar glaciers are subclassified into type I, which present melt water at the base and type II, frozen to their beds. Further, there are polythermal (subpolar) glaciers containing large volumes of ice that are cold, usually present as a surface layer on the ablation area, but also large volumes that are at melting temperature. The last class is temperate glaciers, which are at melting temperature \( \theta_m \) (varies within the glacier) throughout.

To understand how glaciers are formed over time we need to describe the transformation of snow into ice. This transformation is dependent on time and temperature, and varies with different types of glaciers. As presented in [1], the metamorphosis of snow into ice in a glacier is a result of compaction from the overlying layers of snow. The first stage in this process is the diffusion of water molecules from the points of snow flakes toward their centers; flakes tend to become spherical reducing their surface area. Next we have further densification or sintering; in this stage the air space between the particles is reduced and the density increases. This process can be illustrated by referring to the intermediate stages from snow (50-70 km/m\(^3\)) to ice (830-917 km/m\(^3\)), which include damp new snow, settled snow, depth hoar and firn (wetted snow that has survived one summer without being transformed into ice) and have increasing densities in order. An important stage occurs at a density of 830 km/m\(^3\); here firn becomes ice as a result of pores becoming closed which prevents further air movement through the ice.

Within a glacier, ice is zonated with respect to altitude and divided into the following categories: the dry-snow zone, present at high elevations, is the area where no melting occurs during the summer. The percolation zone is at lower elevations and characterized by the fact that the formed during the summer percolates downward into the cold snow where it refreezes. The wet-snow zone stands at lower elevations, and here summer melting is sufficient to wet the entire snow pack; therefore by the end of the summer, all snow deposited since the end of the previous summer has been raised the melting temperature. The superimposed ice zone is at still lower elevations, and here only superimposed ice is present at the end of the melt season. Superimposed ice is characterized by the lack of layers exhibited by the two previous zones, which is due to the high quantities of melt water.

Studying glaciers in space and time involves measuring the changes in their mass, which is defined as mass balance or mass budget studies. These balances are expressed in terms of the thickness of a layer of water or in volumes of water equivalents per unit area. According to this we can give another classification of the zones within a glacier and of the processes by which mass budget changes. Accumulation (area), c, includes all processes by which material is added to the glacier. Ablation (area), a, results from annual melt exceeding snow fall, and therefore all the winter snow and some of the ice melts. Net balance (or budget), bn, is sum of winter balance, bw, and summer balance, bs, over the course of a balance year. A balance year is measured as the time between two minimum thickness values \( t_1 \) and \( t_2 \), and has an average length of 365 days, and \( t_m \) is the the maximum thickness achieved during a balance year [2]. The line separating accumulation and ablation areas is called the equilibrium line. If positive mass balance persists for some years the glacier will advance, and conversely. The formula for calculating accumulation, ablation and net budget are given as:
\[ c = \frac{1}{m} \int_{t_1}^{t_m} c' \, dt \quad a = \frac{1}{m} \int_{t_1}^{t_m} a' \, dt \]

where \( c' \) is the accumulation rate of increase in thickness and \( a' \) ablation rate of decrease in thickness.

\[ bn = bs + bw = \int_{t_1}^{t_m} c' \, dt + \int_{t_1}^{t_m} a' \, dt \]

A number of conditions are used for simplifying physical models of ice. First, ice is considered incompressible, with the density \( \rho \) constant \([1]\). This condition is derived from the conservation of mass and mass flux (rate of mass flow per unit area) in a glacier and is:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]

where \( u, v, w \) are the velocities into the volume in the \( x, y \) and \( z \) directions.

Another assumption is steady-state, namely assuming that the thickness or surface profile doesn't change over a period of years.

Ice flow studies are a major component of ice dynamics. Ice flows by both deformation and basal slip \([1]\). To describe ice deformation, or flow of a crystalline material, we need to study the forces acting in the ice. The forces per unit area are named stresses and they are vectors classified into: normal stresses, directed normal to the surface on which they are acting and shear stresses, directed parallel to the surface. The stress at a point is described by a second-rank tensor composed of 9 stresses, where for each stress vector the 1st subscript identifies the plane on which the stress acts, and the 2nd gives the direction of the stress:

\[
\begin{pmatrix}
\sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\
\sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\
\sigma_{zx} & \sigma_{zy} & \sigma_{zz}
\end{pmatrix}
\]

In order for an object to be in steady uniform motion the forces acting in it must be balanced. Thus, shear forces on any small element must be in equilibrium so that the element does not have a tendency to rotate, and therefore \( \sigma_{yx} = \sigma_{xy} \) with similar relations for the other stresses and the tensor is symmetric \([1]\).

In a deformable medium, stresses induce deformation or strain. Strain is the change, \( \Delta l \), in length of a line divided by the initial length \( l_0 \), denoted \( \varepsilon = \frac{\Delta l}{l_0} \). Strain rate is the rate at which strain occurs, \( \dot{\varepsilon} = \frac{de}{dt} \), \( t \) for time. Some materials, such as ice, present no deformation at stresses below a certain stress, which is the yield stress \([1]\).

Comparing ice with horizontal surface and ice with sloped surface, the first is deformed only by elastic compression, as a result of the action of the hydrostatic pressure \( \rho g z \), while the latter also exhibits pressure differences at points on the same horizontal plane. This pressure difference leads to compressive strain at the different points. To reflect this we define a new set of stresses, deviatoric stresses or non-hydrostatic stresses, \( \sigma'_{xx} = \sigma_{xx} - P \), \( \sigma'_{yy} = \sigma_{yy} - P \), and \( \sigma'_{zz} = \sigma_{zz} - P \) where \( P \) is the mean normal stress

\[ P = \frac{1}{3} (\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) \]

As \( P \) is a normal stress, \( \sigma'_{xy} = \sigma'_{yx} \), and so deformation only depends on the non-hydrostatic components of the stress field since only the normal stresses are different \([1]\).

Considering a plane with angle \( \theta \) with the \( y \) axis which is in static equilibrium we can find \( \sigma_n \) and \( \sigma_s \) by summing the forces acting normal and parallel to the plane and setting this sum to zero. From these we determine the minimum and maximum stresses, called the principal stresses. The orientation and magnitude of the principal stresses is a property of the stress field and does not depend on the coordinate axes chosen, and these are known as invariants of the tensor \([1]\).

Glacier flow involves studying the relationship between the amount/rate of deformation and the applied stress: the constitutive relation. Comparing elastic materials with Newtonian viscous fluids, deformation and stress are proportional in the former whereas rate of deformation is proportional to stress rate in the latter. In perfectly elastic materials a gradual increase in stress produces no deformation until the yield stress is reached, when rapid deformation initiates. For glaciers the constitutive relation is called the flow law, determined experimentally. This is not a universal law, and it depends on conditions such as confining pressure, state of stress and others. Thus ice
responds differently to changes in these conditions: elastic to high-frequency sound waves, intermediate between Newtonian viscous and perfectly elastic if the stress is applied slowly [2]. A generalized flow relation for ice is Glen's flow law:

$$
\varepsilon'_{\text{e}} = \left( \frac{\sigma_{\text{e}}}{B} \right)^n
$$

where B is the viscosity parameter, n empirically determined and \( \sigma_{\text{e}} \) and \( \varepsilon'_{\text{e}} \) are the effective shear stress and the effective strain rate.

The flow field redistributes mass, which determines the shape of a glacier and also redistributes energy which affects temperature distribution. It is fully described by horizontal and vertical components of velocity at every point, given \( b_n(z) \). Thus given accumulation and ablation we use conservation of mass in an incompressible medium to determine mean (averaged over depth) horizontal velocity at some distance \( x \) from the divide [1].

$$
u = \frac{1}{h(x)} \int_0^x b_n(x) \, dx
$$

where \( h(x) \) is the thickness.

By using conservation of momentum and the flow law we can derive horizontal and vertical velocities at the surface and their variation with depth. An important observation is that horizontal velocity in an ice sheet does not increase with depth; the deeper ice moving faster would exert a shear stress on the overlying ice and there would be no opposing forces and so the overlying ice must move at least as fast as that below.

Plotting the velocity vectors at a large number of points we can observe the flow lines in a glacier. Vertical velocities and flow lines tend to be downward in the accumulation area and upward in the ablation area. They are parallel to the bed at the bed and they are parallel to the surface at the equilibrium line. Horizontal velocities increase with the distance from the head of the glacier, reach a maximum just below the equilibrium line and the continue to decrease [1].

Glaciers move over their bed when the average basal temperature is at the pressure melting point determined by the average pressure in the water layer. This movement is depended on the layer of granular rock debris present between the ice or the bed, the till. Till’s deformation is difficult to analyze and varies within the base and across different glaciers. In a simplified study of this process we can consider the movement of clean ice over an irregular hard rigid bed. The processes by which ice moves past the obstacles on a rigid bed are regulation \( S_r \) and plastic flow \( S_p \). Regelation consists of ice melting in the region of high pressure on the upper part of the glacier and refreezing in the region of lower pressure. In a simplified bed geometry model the irregularities in the bed are taken as cubic obstacles of side \( l \) uniformly distributed on a flat surface at distance \( L \) from each other. Plastic flow is deformation of ice in a three dimensional flow field around the obstacle. The sliding speed \( S \) is generally taken as the sum of regelation and plastic flow, \( S = S_r + S_p \). More realistic rough beds can be analyzed [1].

Solutions to the above discussed equations may not always be found using analytical methods, and we have to use numerical methods. Numerical models commonly used are the finite difference and finite element methods. In the case of finite difference models we divide the domain into \( n \) parts of equal length and gradients in a parameter are approximated by evaluating the parameter at grid points and dividing by the distance between the grid points or \( f'(x) = \frac{f(x+h) - f(x)}{h} \) where \( h \) is the length of the division. Computer models using finite difference schemes general divide the domain into small discrete units of equal size; thus \( \Delta z \) should not change with depth and depth is usually normalized by dividing by thickness.

Numerical models of ice sheets usually use the shallow ice approximation. As the horizontal extent of the ice is much larger than its thickness, longitudinal derivatives are small compared to vertical derivatives. If the surface and bed elevations are slowly varying functions of \( x \), the longitudinal coordinate can be scaled according to \( \xi = \mu x \). With this scaling \( \mu \) is introduced in the momentum balance, energy balance and boundary conditions [1].

The viscosity parameter \( B \) is dependent on temperature and the temperature distribution depends on the flow field from the energy balance equation. Ice models must include calculations of both these parameters and these must be done iteratively, and this calculation of energy balance and momentum balance is named thermo mechanical coupling.

**Parallel Ice Sheet Model**

PISM is parallel numerical model of ice sheets developed at the University of Alaska Fairbanks, designed to model the dynamic evolution of ice sheets of different ages and their responses to climatic changes. It simulates a variety of variables, among which are thickness, temperature, velocity, age of the ice and the deformation of the bed. PISM code is C++/C. By default it uses NetCDF for input
and output file format, and requires several libraries such as: GSL (GNU Scientific Library) for numerical calculations and special functions, FFTW (Fastest Fourier Transform in the West) for approximating the deformation of the solid earth under ice, and PETSc (Portable Extensible Toolkit for Scientific computation) which uses MPI (Message Passing Interface).

PISM was downloaded to Max via subversion using the command 'svn co http://svn.gna.org/svn/pism/trunk pism'. MPI and PETSc were installed onto the local directory as they are prerequisites. Initially, the versions of MPI and PETSc already installed in Max were used but their MPI was non-functional and thus, all PISM requirements were installed manually to avoid such issues. Chris Grim of Max administration and Jed Brown, now at ETH Switzerland, were very helpful during the installation process. Verification against the solutions of the continuum model is built-in.

We tested PISM on the EISMINT II simplified-geometry, thermo mechanically-coupled ice sheet model intercomparison and also on real data obtained from the Ross Ice Shelf in Antarctica. For any run, PISM outputs a summary of the model state using a few variables at each time step [3]:

$$\text{YEAR} (+ \text{STEP}[N]) \quad \text{VOL} \quad \text{AREA} \quad \text{MELTF} \quad \text{THICK0} \quad \text{TEMP0}$$

The first five symbols tell the user which quantities are being updated at that time step, and are replaced by a dollar sign if the quantity is not updated. The variables updated are the following: [b$] bed elevation, [vV$] velocity (the lower case “v” indicates the update in only the vertically-averaged velocity), [g$] grain size, [t$] temperature and age (updated together), and [f$] surface elevation. The time (“YEAR”) and time step (“STEP”) are in years; STEP represents the time step just taken by PISM determined by adaptive time stepping mechanism. This is indicated by the [N$] next to the STEP column; examples of time steps chosen are “e”, the time step was shortened to hit the end of the specified run and “m” the step was the maximum allowed, namely 60 years.

The following columns are the volume VOL of the ice in $10^6$ km$^3$, the area AREA covered by the ice in $10^6$ km$^2$, the basal melt fraction MELTF (fraction of the ice area where the basal temperature is above 273.0K, slightly lower than the triple point), the thickness THICK0 in meters and the basal absolute temperature TEMP0 in Kelvin.

The model can output at runtime a larger summary of variables, which include: d(volume)/dt of ice (km$^3$/a), average value of dH/dt (m/a), area percent covered by ice, max $|u|$, $|v|$, $|w|$ in ice (m/a), fraction of the ice which is original and others.

PISM does all simulations in a rectangular computational box. The coordinates are x, y in the horizontal and z in the vertical, which is measured positive upward from the base of the ice. z = 0 is the base of the ice in both the grounded and floating cases and bedrock corresponds to negative z values. The coordinate grid is equally spaced in each direction, which can be x direction, y direction, z direction into the ice and z direction into the bedrock.

**Simplified Geometry Experiments (SGE) model intercomparison**

Since the 1990s the European Science Foundation has organized the European Ice Sheet Model Initiative, or EISMINT benchmarks, for testing and intercomparison of numerical ice sheet model based on simplified geometry experiments [6]. This occurred in two stages, namely EISMINT I experiments, which were not included in PISM and EISMINT II which we will be presenting.

EISMINT II consists of 7 experiments, A, B, C, D, E, F, G, H, which share common features. The experiments are 200,000 years runs. Since the equilibrium is achieved after 50,000 years, we used this value for each of our runs. The prescribed horizontal grid has 60 intervals in both x and y directions, each of 25 km, with a 1500 km total width of computational box; the vertical grid not prescribed and all three grids can be changed at runtime. PISM developers recommend 201 grid points in the vertical for a 25 m (equally-spaced) grid, so we will always be using a grid 61x61x201 grid [3]. Only very simplified, regular boundary conditions are used, i.e. simplified geometry and the ice sheet is symmetric, circular and grounded on a flat bed which does not move. The temperature in the bedrock is not modelled. Only SIA is included and thermo mechanical coupling is included.

I will present experiments A, B, C, D and F. Each experiment used the specification of A and changes a single input parameter to observe the effects. Experiment A is the initial thermo mechanical coupling run. A and F start with zero ice, but F has a lower minimum surface temperature by 15K. Experiments B, C and D start from the final state of experiment A. Compared to experiment A, B has a 5K higher maximum surface temperature, C has a reduced maximum accumulation rate and also a reduced area over which this operates (ice sheet span), and D has only a smaller area of accumulation or ice sheet span [5].

The default parameters for these experiments are:

- Mmax, max value of accumulation rate in m/a: 0.5 for A, B, D, F, and 0.25 for C
- Rel, radial distance to equilibrium line in km: 450 for A, B, F and 425 for C, D
- Sb, radial gradient of accumulation rate change with horizontal distance in (m/a)/km: 10^{-2} for all
- ST, radial gradient of air-temperature change with horizontal distance in K/km: 1.67* 10^{-2} for all
• Tmin, minimum of surface air temperature in K: 238.15 for A, C, D, 243.15 for B and 223.15 for F
• height of the computational box in m: 4500 for A, 4000 for B, C, D and 5000 for F
• \((x_{\text{summit}}, y_{\text{summit}})\) in km: 750

All experiments predict a central zone frozen to the bed and an outer zone whose base is at the pressure melting point. The major finding, as described in [5], is that the radial symmetry implied by the experiments is replaced by spokes of cold ice which extend outward from the center of the ice sheet. These spokes can also appear in the thickness and velocity distributions. The radial symmetry of the experiments is implied by the ice accumulation/ablation rate in m/a and the ice-surface temperature in K. All other conditions are either constant or also radially symmetric: mass balance pattern and others. For simplification, both these equations are dependent only on geographical position and not on elevation.

\[
M(x, y) = \min[M_{\text{max}}, S_b(Re l - \sqrt{(x-x_{\text{summit}})^2 + (y-y_{\text{summit}})^2})]
\]

\(M(x,y)=0\) at Rel distance from the summit \((x_{\text{summit}}, y_{\text{summit}})\), and this parameterization results in a large accumulation area; around this area \(M\) decreases linearly with distance from the summit and becomes negative.

\[
T_{\text{surface}}(x, y) = T_{\text{min}} + ST \times \sqrt{(x-x_{\text{summit}})^2 + (y-y_{\text{summit}})^2}
\]

For each experiment we will present the summary variables outputted by PISM at each time step, for a few time steps chosen randomly, including the first and last years. Next we include map-plane views of surface altitude, land ice thickness, thickness of the subglacial melt water, mean ice accumulation rate, land ice temperature, bedrock temperature and surface temperature along with their time series graph. Time series plot was started from center of the map plane view and continued going up horizontally.

1. Experiment A
   a. No changes in the initial parameters

<table>
<thead>
<tr>
<th>YYYY</th>
<th>YEAR (+ STEP[NS]):</th>
<th>VOL</th>
<th>AREA</th>
<th>MELTF</th>
<th>THICK0</th>
<th>TEMP0</th>
</tr>
</thead>
<tbody>
<tr>
<td>60.000</td>
<td>60.000000[0m]:</td>
<td>0.017</td>
<td>0.628</td>
<td>0.000</td>
<td>30.000</td>
<td>238.150</td>
</tr>
<tr>
<td>9587.008</td>
<td>1.08209[0d]:</td>
<td>2.310</td>
<td>0.991</td>
<td>0.323</td>
<td>4000.998</td>
<td>250.962</td>
</tr>
<tr>
<td>21054.869</td>
<td>0.97694[0d]:</td>
<td>2.211</td>
<td>1.036</td>
<td>0.425</td>
<td>3678.794</td>
<td>254.401</td>
</tr>
<tr>
<td>49998.416</td>
<td>1.10038[0d]:</td>
<td>2.144</td>
<td>1.036</td>
<td>0.401</td>
<td>3630.663</td>
<td>255.705</td>
</tr>
<tr>
<td>50000.000</td>
<td>0.48393[0e]:</td>
<td>2.144</td>
<td>1.036</td>
<td>0.401</td>
<td>3630.648</td>
<td>255.705</td>
</tr>
</tbody>
</table>

Surface altitude \(h\), ranging from 0 to 3630.65 m
Land ice thickness $H$, ranging from 0 to 3630.65 m

Thickness of sub glacial melt water $H_{melt}$, ranging from 0 to 5m

Mean Ice equivalent accumulation rate $\text{accum}$, ranging from -1.95311e-07 to 1.58444e-08 m/s
Land ice temperature $T$, ranging from 238.15 to 272.064 K

Bedrock temperature $T_b$, ranging from 238.15 to 272.064 K

Surface temperature $T_s$, ranging from 238.15 to 255.86 K
b. This experiment is a variation of A, with the increase from 0.5 to 0.25 in the maximum accumulation rate Mmax

<table>
<thead>
<tr>
<th>SvStf</th>
<th>60.000 (+ 60.000000 [0m]):</th>
<th>0.009</th>
<th>0.628</th>
<th>0.000</th>
<th>15.000</th>
<th>238.150</th>
</tr>
</thead>
<tbody>
<tr>
<td>SvStf</td>
<td>20926.070 (+ 1.62825 [0d]):</td>
<td>1.950</td>
<td>0.921</td>
<td>0.277</td>
<td>3449.534</td>
<td>256.269</td>
</tr>
<tr>
<td>SvStf</td>
<td>20927.699 (+ 1.62829 [0d]):</td>
<td>1.950</td>
<td>0.921</td>
<td>0.277</td>
<td>3449.485</td>
<td>256.270</td>
</tr>
<tr>
<td>SvStf</td>
<td>39028.228 (+ 1.57277 [0d]):</td>
<td>1.832</td>
<td>0.921</td>
<td>0.263</td>
<td>3305.441</td>
<td>258.753</td>
</tr>
<tr>
<td>SvStf</td>
<td>49999.477 (+ 1.80596 [0d]):</td>
<td>1.814</td>
<td>0.921</td>
<td>0.250</td>
<td>3290.863</td>
<td>259.276</td>
</tr>
<tr>
<td>SvStf</td>
<td>50000.000 (+ 0.52276 [0e]):</td>
<td>1.814</td>
<td>0.921</td>
<td>0.250</td>
<td>3290.864</td>
<td>259.276</td>
</tr>
</tbody>
</table>

The surface altitude h, ranging from 0 to 3290.85 m, the land ice thickness H, ranging from 0 to 3290.85 m and the mean ice equivalent accumulation rate accum, ranging from -1.95311e-07 to 7.92219e-09 m/s and the surface temperature Ts, ranging from 238.15 to 255.86 K present the same map plane views as Experiment A, and therefore we did not include them.

Thickness of sub glacial melt water Hmelt, ranging from 0 to 5 m

Land ice temperature T, ranging from 238.15 to 272.085 K
Bedrock temperature $T_b$, ranging from 247.163 to 272.085 K

$T_b$ values:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
<th>$T_b$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>243.150</td>
</tr>
<tr>
<td>60.000</td>
<td>60.000</td>
<td>0.000</td>
<td>243.150</td>
</tr>
<tr>
<td>5151.222</td>
<td>7.82502</td>
<td>0.000</td>
<td>252.377</td>
</tr>
<tr>
<td>5163.480</td>
<td>12.25712</td>
<td>0.000</td>
<td>252.389</td>
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<tr>
<td>49999.078</td>
<td>1.13450</td>
<td>0.000</td>
<td>252.377</td>
</tr>
<tr>
<td>50000.000</td>
<td>0.92239</td>
<td>0.000</td>
<td>252.377</td>
</tr>
</tbody>
</table>

Surface altitude $h$, ranging from 0 to 3314.7 m, land ice thickness $H$, ranging 0 to 3314.7 m, mean ice equivalent accumulation rate $\text{accum}$, ranging from $-1.95311 \times 10^{-7}$ to $1.58444 \times 10^{-8}$ m/s and surface temperature $T_s$, ranging from 243.15 to 260.863 K present the same map plane views as Experiment A, and therefore we did not include them.

Thickness of subglacial melt water $H_{\text{melt}}$, ranging from 0 to 5 m
Land ice temperature $T$, ranging from 248.15 to 272.247 K

Bedrock temperature $T_b$, ranging from 252.753 to 272.247 K

d. Variation of $\alpha$, with changes in maximum accumulation rate from 0.5 to 0.75 m/a, and in the height of the computational box to 5000 m

\[ \text{YEAR (+ STEP[NS]): VOL AREA MELTF THICK0 TEMP0} \]
\[ \text{SvStf 60.000 (+ 60.00000[0m]): 0.024 0.628 0.000 45.000 238.150} \]
\[ \text{SvStf 49999.709 (+ 0.87237[0d]): 2.379 1.113 0.503 3812.113 253.872} \]
\[ \text{SvStf 50000.000 (+ 0.29124[0e]): 2.379 1.113 0.503 3812.118 253.872} \]

Surface altitude $h$, ranging from 0 to 3812.118 m, land ice thickness $H$, ranging from 0 to 3812.118 m, mean ice equivalent accumulation rate $\text{accum}$, ranging from $-1.95311e-07$ to $2.3766e-08$ m/s and surface temperature $T_s$, ranging from 228.15 to 255.863 K present the same map plane views as Experiment A, and therefore we did not include them.
Thicknes of subglacial melt water $H_{\text{melt}}$, ranging from 0 to 5m

Land ice temperature $T$, ranging from 238.15 to 272.021 K

Bedrock temperature $T_b$, ranging from 248.109 to 272.021 K
2. Expriment B
   a. No changes in the initial parameters

   $SSSSS \ 50000.000 \ (+ \ 0.00000[0d]): \ 2.144 \ 1.036 \ 0.401 \ 3630.648 \ 255.705$
   $SvStf \ 66423.137 \ (+ \ 1.23841[0d]): \ 1.982 \ 1.036 \ 0.543 \ 3397.523 \ 257.287$
   $SvStf \ 76237.747 \ (+ \ 1.10266[0d]): \ 1.942 \ 1.031 \ 0.565 \ 3353.295 \ 259.048$
   $SvStf \ 82060.959 \ (+ \ 1.19398[0d]): \ 1.939 \ 1.026 \ 0.563 \ 3353.571 \ 259.536$
   $SvStf \ 99999.555 \ (+ \ 1.08991[0e]): \ 1.938 \ 1.026 \ 0.570 \ 3343.525 \ 260.013$
   $SvStf \ 100000.000 \ (+ \ 0.44503[0e]): \ 1.938 \ 1.026 \ 0.570 \ 3343.518 \ 260.013$

   Surface altitude $h$, ranging from 0 to 3343.52 m, land ice thickness $H$, ranging from 0 to 3343.52 m, mean ice equivalent accumulation rate $\text{accum}$, ranging from $-1.95311e-07$ to $1.58444e-08$ m/s and surface temperature $T_s$, ranging from 243.15 to 260.863 K present the same map plane views as Experiment A, and therefore we did not include them.

   Thickness of subglacial melt water $H_{melt}$, ranging from 0 to 5 m

   Land ice temperature $T$, ranging from 243.15 to 272.269 K
Bedrock temperature $T_b$, ranging from 252.68.15 to 272.269 K

b. Variation of A, started from A with accumulation rate 0.25m/a, continuing with the same accumulation rate of 0.25m/a

Surface altitude $h$, ranging from 0 to 2999.358 m, land ice thickness $H$, ranging from 0 to 2999.358 m, mean ice equivalent accumulation rate $\text{accum}$, ranging from $-1.95311\times10^{-7}$ to $7.92219\times10^{-9}$ m/s and surface temperature $T_s$, ranging from 243.15 to 260.863 K present the same map plane views as Experiment A, and therefore we did not include them.

Thickness of subglacial melt water $H_{melt}$, ranging from 0 to 5m
Land ice temperature \( T \), ranging from 243.15 to 272.212 K

Bedrock temperature \( T_b \), ranging from 248.109 to 272.021 K

3. Experiment C

\[
\text{Experiment C}
\]

\[
\text{YEAR} (+ \text{STEP}[NS]) : \text{VOL AREA MELTF THICK0 TEMP0}
\]

\[
\text{50000.000 (+ 0.00000[0])} : 2.144 \ 1.036 \ 0.401 \ 3630.648 \ 255.705
\]

\[
\text{72876.447 (+ 1.58594[0d])} : 1.545 \ 0.813 \ 0.286 \ 3223.846 \ 258.322
\]

\[
\text{90428.451 (+ 2.00266[0d])} : 1.534 \ 0.813 \ 0.289 \ 3187.028 \ 259.126
\]

\[
\text{99999.711 (+ 1.84123[0d])} : 1.535 \ 0.813 \ 0.295 \ 3192.936 \ 259.310
\]

\[
\text{100000.000 (+ 0.28953[0e])} : 1.535 \ 0.813 \ 0.295 \ 3192.935 \ 259.310
\]

Mean Ice equivalent accumulation rate \( \text{accur} \), ranging from \(-2.01433e-07\) to \(7.92219e-09\) m/s and surface temperature \( T_s \), ranging from 238.15 to 255.863 K present the same map plane views as Experiment A, and therefore we did not include them.
Surface altitude $h$, ranging from 0 to 3192.94 m

Land ice thickness $H$, ranging from 0 to 3192.94 m

Thickness of subglacial melt water $H_{\text{melt}}$, ranging from 0 to 5 m
Land ice temperature $T$, ranging from 238.15 to 272.385 K

Bedrock temperature $T_b$, ranging from 246.384 to 272.385 K

4. Experiment D

Surface altitude $h$, ranging from 0 to 3519.82 m, land ice thickness $H$, ranging from 0 to 3519.82 m, mean ice equivalent accumulation rate $\text{accum}$, ranging from $-2.01433e^{-07}$ to $1.5844e^{-08}$ m/s and surface temperature $T_s$, ranging from 238.15 to 255.863 K present the same map plane views as Experiment A, and therefore we did not include them.
Thickness of subglacial melt water $H_{\text{melt}}$, ranging from 0 to 5 m.

Land ice temperature $T$, ranging from 238.15 to 272.326 K.

Bedrock temperature $T_b$, ranging from 247.306 to 272.326 K.
5. Experiment F

a. No changes in the initial parameters

\begin{align*}
\text{SvStf} & \quad 60.000 (+ 60.00000[\text{m}]): \quad 0.017 \quad 0.628 \quad 0.000 \quad 30.000 \quad 223.150 \\
\text{SvStf} & \quad 5971.674 (+ 5.68703[\text{c}]): \quad 1.690 \quad 0.693 \quad 0.018 \quad 2985.837 \quad 233.121 \\
\text{SvStf} & \quad 10857.534 (+ 0.86149[\text{d}]): \quad 2.758 \quad 0.938 \quad 0.112 \quad 4966.166 \quad 236.861 \\
\text{SvStf} & \quad 20379.312 (+ 0.70359[\text{d}]): \quad 2.758 \quad 1.043 \quad 0.242 \quad 4489.121 \quad 240.203 \\
\text{SvStf} & \quad 30968.703 (+ 0.76453[\text{d}]): \quad 2.666 \quad 1.043 \quad 0.220 \quad 4428.943 \quad 241.536 \\
\text{SvStf} & \quad 40049.690 (+ 1.03449[\text{d}]): \quad 2.641 \quad 1.038 \quad 0.212 \quad 4407.985 \quad 242.009 \\
\text{SvStf} & \quad 45462.087 (+ 1.08398[\text{d}]): \quad 2.631 \quad 1.038 \quad 0.214 \quad 4404.384 \quad 242.143 \\
\text{SvStf} & \quad 50000.000 (+ 0.76290[\text{e}]): \quad 2.623 \quad 1.031 \quad 0.376 \quad 4403.738 \quad 242.377
\end{align*}

Surface altitude h, ranging from 0 to 4400.6 m same as A, land ice thickness H, ranging from 0 to 4400.6 m, mean ice equivalent accumulation rate \text{accum}, ranging from \(-2.01433e-07\) to \(1.5844e-08\) m/s and surface temperature \text{Ts}, ranging from 223.15 to 240.863 K present the same map plane views as Experiment A, and therefore we did not include them.

Thickness of subglacial melt water \text{Hmelt}, ranging from 0 to 5 m

\begin{figure}
\centering
\includegraphics[width=\textwidth]{thick_of_subglacial_melt_water.png}
\caption{Thickness of subglacial melt water}
\end{figure}

Land ice temperature T, ranging from 223.15 to 271.903 K

\begin{figure}
\centering
\includegraphics[width=\textwidth]{land_ice_temperature.png}
\caption{Land ice temperature}
\end{figure}
Bedrock temperature $T_b$, ranging from 232.689 to 271.903 K

b. Variation of $F$, change in $T_{\text{min}}$ to 228.15

$S_v$ = 60.000 (+ 0.00000[0m]): 0.017 0.628 0.000 30.000 228.150
$S_v$ = 49999.343 (+ 0.96555[0d]): 2.479 1.033 0.254 4152.553 246.772
$S_v$ = 50000.000 (+ 0.65743[0e]): 2.479 1.033 0.254 4152.543 246.772

Surface altitude $h$, range 0 to 4152.54 m, land ice thickness $H$, range 0 to 4152.54.7 m and mean ice equivalent accumulation rate $\text{accum}$, range -1.93511e-08 to 1.5844e-08 m/s present the same map plane views as Experiment A, and therefore we did not include them.

Thickness of subglacial melt water $H_{\text{melt}}$, ranging from 0 to 5m
Land ice temperature $T$, ranging from 228.15 to 271.875 K

Bedrock temperature $T_b$, ranging from 237.68 to 271.875 K

**Realistic ice sheet and ice shelf modelling**

Validation consists of comparison of the model with physical observations, and gives information of the quality of the model. We will validate PISM relative to the Ross Ice Shelf data, as done in the EISMINT-Ross series of intercomparisons [4]. The comparison is done relative to the RIGGS data (Ross Ice Shelf Geophysical and Glaciological Survey) acquired in 1973-1978. RIGGS data includes the (horizontal) velocity of the ice shelf measured at a few hundred locations in a regular grid across the shelf.

After running the validation we can choose to output the result in Matlab and produce chi square statistic relative to RIGGS data and the respective graphics. This gives $\text{ChiSqr} = 2954.0$ and $\text{max\_computed\_speed} = 1149.5$
This can results can be compared to the Ross Ice Shelf intercomparison in [4].

**Results and Conclusions**

For each of the experiments presented, PISM outputs the temperature distribution, the velocity field, map plane views of surface altitude, land ice thickness, mean ice equivalent accumulation rate, thickness of sub glacial melt, land ice temperature and bedrock temperature. For a number of the EISMINT II experiments, spokes of cold ice which extend outward from the center of the ice sheet appear in the basal temperature field as well as in the thickness distribution, and this is an interesting and unexpected result.

All the experiments are a variation on the initial conditions of experiment A. A presents spokes in the bedrock temperature, land ice temperature and thickness. Changing the maximum accumulation rate in A from 0.5 m/a to 0.25 m/a gives a very different thickness profile and the spokes are no longer present in any field. Increasing the temperature from 238.15K to 243.15K or increasing the accumulation rate to 0.75 m/a and the computational box to 5000m gives an output fairly similar to A.

Experiment B starts at the end of A with a temperature 5K higher, and is also quite similar to A. By continuing B from the end of A with 0.25 m/a maximum accumulation rate, and keeping this value constant we see that some spokes are starting to appear, but the do not go very deep into the center of the ice. Experiment C starts from the end of A with, and has a reduced maximum accumulation rate and also a reduced area over which this operates and experiment D has only a smaller area of accumulation or ice. We can see that these changes don't influence the behavior of the ice sheet very much, as the map plane pictures are similar to A, although the spokes are no longer identical. Lastly experiment F starts with zero ice and temperature lower by 15K. Here the temperature distributions exhibit a much larger number of spokes. If we further lower the temperature by 10K we observe that this gives some stability, as the number of spokes gets reduced.

PISM was a means for me to combine learning theoretical ice dynamics and watching a model ice sheet evolve over time. The model was used on a number of benchmark experiments, EISMINT II, and also on real ice sheet data, the Ross Ice Shelf. By running various tests and sensitivity experiments on PISM I was able to verify the ice dynamics discussed above.
References:


